Total No. of printed pages = 8

MATHEMATICS – II

(New Course)

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

PART – A

Marks – 25

1. Find the correct answer : $1\times10=10$

(i) Radius of the circle $x^2+y^2-6x+2y+1=0$ is

(a) 4 (b) 6

(c) 3 (d) 11

(ii) Equation to the normal to the circle $x^2 + y^2 = 9$ at (3, 0) is

(a) $y = 2$ (b) $y = 0$

(c) $x = 0$ (d) $x = 1$

(iii) Minor axis of the ellipse $x^2 + 9y^2 = 144$ is

(a) 5 (b) 3

(c) 8 (d) 9

(iv) Focus of the parabola $y^2 = 12x$ is

(a) (4, 0) (b) (–3, 0)

(c) (3, 0) (d) (0, – 3)

(v) Direction ratios of the line joining (2, 3, – 1) and (0, 4, 1) are

(a) 2, 1, 6

(b) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(c) 4, 2, 1

(d) 2, –1, –2

(vi) Angle between the lines with direction ratios – 1, 2, 3 and 5, 1, 1 is

(a) 45° (b) 60°

(c) 90° (d) 30°

(vii) Distance between (2, 1, 0) and (0, −1, 1) is

(a) 3 (b) $\sqrt{5}$

(c) 1 (d) 11

(viii) Cross product of $2\hat{i}+\hat{j}-4\hat{k}$ and $5\hat{i}-2\hat{j}+\hat{k}$ is

(a) $\hat{i}+3\hat{j}+5\hat{k}$ (b) $6\hat{i}+11\hat{j}+2\hat{k}$

(c) $\hat{i}+\hat{j}-2\hat{k}$ (d) $-7\hat{i}-22\hat{j}-9\hat{k}$

(ix) Dot product of $4\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-2\hat{j}+9\hat{k}$ is

(a) 27 (b) –27

(c) – 4 (d) 0

(x) If $\vec{a}=\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b}=5\hat{i}-2\hat{j}-\hat{k}$ then $|\vec{a}-\vec{b}|$ is

(a) 13 (b) 12

(c) 5 (d) 4

2. Fill in the blanks : $1\times10=10$

(i) Domain of the real valued function $f(x)=\frac{3}{x-1}$ is ———.

(ii) Domain of the real valued function $f(x)=\sqrt{x^2-9}$ is ———.

(iii) Value of $\lim\limits_{x\to0}\frac{\sin^2 4x}{x}$ is ———.

(iv) Value of $\lim\limits_{x\to\infty}\frac{1}{x^3-1}$ is ———.

(v) Value of $\lim\limits_{x\to2}\frac{x^2-3x+2}{x^2-4}$ is ———.

(vi) Derivative of $y = 5x - 2$ with respect to x is ———.

(vii) Derivative of $y = e^{4x}$ with respect to x is ———.

(viii) Derivative of $y = \log x^4$ with respect to x is ———.

(ix) Second order derivative of $s = t^3 + 5 \cos t + 4$ at $t = 0$ is ——.

(x) Second order derivative of $y = \tan^{-1} 2x$ at $x = 1$ is ——.

3. Write true or false : $1 \times 5 = 5$

(i) $\int \sin x \, dx = \cos x$

(ii) $\int 4x^3 dx = x^4$

(iii) $\int \sqrt{a^2 - x^2} \, dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a}$

(iv) $\int 2 \sin 2x \, dx = - \cos 2x$

(v) Area bounded by the parabola $y^2 = 4x$ and its latus rectum is $\frac{3}{8}$.

PART – B

Marks – 45

4. Write center and diameter of the circle $x^2 + y^2 + 8x + 2y + 13 = 0$. 2

5. Answer any *three* : 3×3=9

(i) Find the equation of the circle having centre (2, 1) and which passes through (0, 0). Find its radius.

(ii) Trace the parabola $x^2 = 5y$.

(iii) Find the equation to the tangent to the ellipse $9x^2 + 4y^2 = 25$ at $(-1, -2)$.

(iv) Find unit vector perpendicular to $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 5\hat{j} + \hat{k}$.

(v) Find the projection of the line joining $(-2, 3, -1)$ and $(3, 4, 3)$ on the line joining the points $(3, 5, 2)$ and $(5, 3, 3)$.

6. (a) Find limit (any *two*) : 5×2=10

(i) $\lim\limits_{x \to 0} \dfrac{\sin 4x}{\sin 3x}$

(ii) $\lim\limits_{x \to \infty} \dfrac{2x^3 - 1}{2x^3 + 5x^2 + 1}$

(iii) $\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$

(iv) $\lim_{x \to 0} \frac{e^{2x} - e^{3x}}{x}$

(b) Find $\frac{dy}{dx}$ (any *three*) : $3 \times 3 = 9$

(i) $y = 3x^5 + \sqrt{x}$

(ii) $y = \sqrt{(x-1)^3 + 2}$

(iii) $y = \log \tan x$

(iv) $x = \sin^2 \theta$, $y = \tan \theta$

(v) $x^y y^x = 1$

7. Find $\frac{a^2 y}{ax^2}$ of $y = \tan^{-1} 2x$ 3

8. Find maximum and minimum value of $y = \sin x$. 3

9. Evaluate (any *three*) : 2×3=6

(i) $\int \frac{2}{x^3}\, dx$

(ii) $\int \left(x^2 + e^x - \sin 4x\right) dx$

(iii) $\int \frac{e^x dx}{e^{2x} + 1}$

(iv) $\int \frac{dx}{9 + x^2}$

(v) $\int \sin 2x \cos 3x \, dx$

(vi) $\int_0^{10} e^{2x} dx$

10. Find area bounded by $y = 2x^3$, $x = 3$ and x-axis. 3